An Efficient Conformal LOD-FDTD Method and its Numerical Dispersion Analysis

Xiao-Kun Wei[1], Wei Shao[1], *Member, IEEE* and Sheng-Jun Zhang[2]

[1]School of Physical Electronics, University of Electronic Science and Technology of China, Chengdu, China
weixiaokun1990@163.com, weishao@uestc.edu.cn
[2]National Key Laboratory of Science and Technology on Test Physics & Numerical Mathematics, Beijing, China
zhangsj98@sina.com.cn

In this paper, an efficient conformal locally one-dimensional finite-difference time-domain (LOD-CFDTD) method is proposed and its theoretical numerical dispersion analysis is presented. Instead of staircasing approximation, the conformal scheme is only employed to model the curved boundaries, whereas the standard Yee grids are used for the remaining regions. As the irregular grids accounts for a very small percentage of the total space grids, the conformal scheme has little effect on the numerical dispersion. With the total-field/scattered-field (TF/SF) boundary and perfectly matched layer (PML), the radar cross section (RCS) of a 2-D structure is calculated to verify the accuracy and efficiency of the LOD-CFDTD method.

***Index Terms*—Conformal, locally one-dimensional (LOD) finite-difference time-domain (FDTD), numerical dispersion analysis, scattering.**

I. Introduction

The finite-difference time-domain (FDTD) method based on the locally one-dimensional (LOD) scheme eliminates the temporal stability constraint of conventional FDTD [1]. However, although the time step in the LOD-FDTD method is not limited by the Courant-Friedrich-Levy (CFL) condition, a large time step will lead to large numerical dispersion errors [2]. Some improved schemes have been introduced to reduce the numerical dispersion error and enhance the efficiency of the conventional LOD-FDTD method [3-5]. Recently, a nonorthogonal LOD-FDTD (LOD-NFDTD) method was proposed for scattering problems in the curvilinear coordinate system and high performance was achieved [6], but it can be seen that its implementation is relatively complex. An easily implemented conformal scheme introduced to LOD-FDTD (LOD-CFDTD) can be used to reduce the staircasing error, in which the conformal grids are only applied to the curved metallic boundary, whereas standard Yee grids are used in the remaining computational regions. In this paper, the efficient LOD-CFDTD method is presented and its numerical dispersion is analytically analyzed.

II. LOD-CFDTD Method and its Numerical Dispersion

On the irregular grids intersected by a perfect electric conductor (PEC), the E-field is updated exactly in the same way as in the conventional LOD-FDTD method, while the H-field is updated in a different way based upon Faraday's law along the circumference of the irregular grids. Following the derivation of LOD-FDTD's formulation [1], the two sub-steps of LOD-CFDTD that convert the explicit scheme into an implicit one can be obtained.

Starting from the time-domain Maxwell's equations for a two-dimensional (2-D) TE_z wave and Fourier analysis for a monochromatic wave, the numerical dispersion relation of the LOD-CFDTD method can be derived as follows [2]:

$$\frac{\varepsilon\mu}{\Delta t^2}\sin^2\left(\frac{\omega\Delta t}{2}\right)=\frac{\Delta y'}{\Delta x S}\sin^2\left(\frac{k_x\Delta x}{2}\right)\left(\frac{e^{j_0\frac{\omega\Delta t}{2}}+1}{2}\right)^2+\frac{\Delta x'}{\Delta y S}\sin^2\left(\frac{k_y\Delta y}{2}\right)\left(\frac{e^{-j_0\frac{\omega\Delta t}{2}}+1}{2}\right)^2 \tag{1}$$

where $j_0=\sqrt{-1}$, $\Delta x'$ and $\Delta y'$ are the lengths of an irregular grid outside the PEC along x- and y- directions, respectively, and S is the area of the irregular grid outside the PEC.

It is worth mentioning that (1) will be converted into the numerical dispersion of LOD-FDTD in regular grid when $\Delta x'=\Delta x$ and $\Delta y'=\Delta y$. Obviously, as Δt, Δx and Δy tend to zero, (1) will be converted into the ideal dispersion relation

$$k_x^2+k_y^2=(\omega/c)^2=k^2 \tag{2}$$

where $k = \omega/c$ is the theoretical wave-number and c is the speed of light in free space.

To study the effect on numerical dispersion from different irregular grids, the ratio R_i is defined as

$$R_i=\frac{1}{2}\frac{\Delta x'|_i\,\Delta y}{S|_i}+\frac{1}{2}\frac{\Delta y'|_i\,\Delta x}{S|_i}. \tag{3}$$

It is the fact that the conformal scheme with different R_i has much effect on numerical dispersion for a single irregular grid. An example of a 2-D circular waveguide with a radius of 4.5mm is used to validate the numerical dispersion in the whole computational region. The weighted R in the whole computational region is calculated by

$$R=\frac{\sum_{i=1}^{N}R_i}{N} \tag{4}$$

where N is the total number of grids. The normalized phase velocity with propagation direction of LOD-CFDTD is shown in Fig. 1. It can be seen that a small percentage of irregular grids contributes little effect on numerical dispersion in the whole computational region.

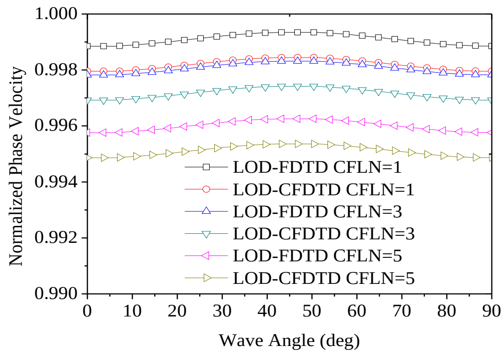

Fig. 1. Normalized phase velocity vs. the wave angle from LOD-FDTD and LOD-CFDTD with different CFLNs (CFL numbers) and PPW (points per wavelength) = 40.

To validate the accuracy of proposed method for different PPWs and CFLNs, the cutoff frequency of TE_{11} mode is calculated in the circular waveguide. Fig. 2 shows the relative errors obtained by LOD-FDTD using staircasing approximation and LOD-CFDTD with PPW = 20 (where R = 0.9667) and 40 (where R = 0.9982) at different CFLNs, respectively. It can be seen that the results obtained from the proposed LOD-CFDTD method are more accurate than the staircase approximation scheme with the same PPW.

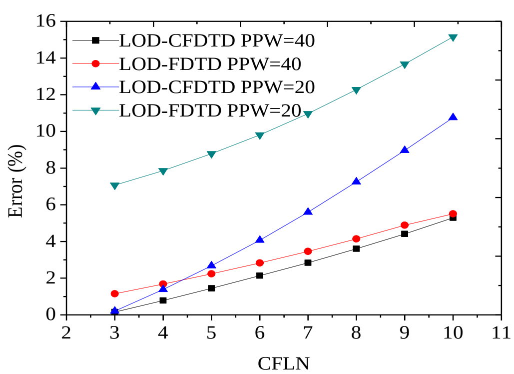

Fig. 2. Temporal discretization convergence curves of the LOD-FDTD and LOD-CFDTD for the 2-D PEC circular waveguide.

III. Numerical Result

In order to validate the accuracy and efficiency of the proposed LOD-CFDTD method, the scattering by two PEC elliptic cylinders which are infinitely long in the z-direction is computed in this section. Fig. 3 displays the configuration of the two structures, where $a = \lambda/10$, $b = 2\lambda$ and $s = 0.4\lambda$.

Here, results obtained from both the CFDTD and alternating-direction-implicit (ADI) CFDTD methods are also shown for comparison. The bistatic RCS for the two 2-D PEC elliptic cylinders is shown in Fig. 3. Obviously, the results obtained with LOD-CFDTD are in good agreement with those obtained from CFDTD and ADI-CFDTD.

A comparison between the CFDTD, ADI-CFDTD and LOD-CFDTD methods in terms of execution time and memory usage for calculating the RCS of this example is shown in Table I. With the same grids density, the CPU time for LOD-CFDTD is less than CFDTD and ADI-FDTD while maintaining the acceptable accuracy. The calculation has been performed on an AMD Phenom II×4 3.0 GHz machine with 4 GB RAM.

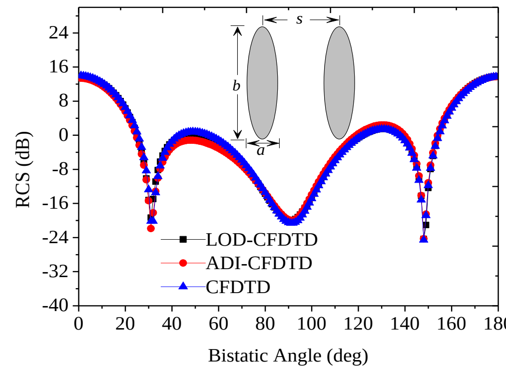

Fig. 3. Bistatic RCS of two PEC elliptic cylinders from three methods.

TABLE I
Comparison of Computer Resources for the Two PEC Elliptical Cylinders Using CFDTD, ADI-CFDTD and LOD-CFDTD

Methods	Δt (ps)	Marching steps	CPU time (s)	Memory (MB)
CFDTD	5.00	12000	1726.79	6.78
ADI-CFDTD	15.00	4000	1459.99	12.11
LOD-CFDTD	15.00	4000	1205.38	11.33

IV. Conclusion

In this paper, an efficient LOD-CFDTD method is presented and its numerical dispersion is analytically analyzed. It is proved that the conformal grids which account for a small percentage of the whole grids have little effect on its numerical dispersion, and the numerical result validates the accuracy and effectiveness of the proposed method.

Acknowledgement

This work was supported by the National Natural Science Foundation of China (No. 61331007, 61471105 and 61172019).

References

[1] J. Shibayama, M. Muraki, J. Yamaucyhi, and H. Nakano, "Efficient implicit FDTD algorithm based on locally one dimensional scheme," *Electron. Lett.*, vol. 41, no. 19, pp. 1046-1047, Sep. 2005.

[2] I. Ahmed, E. K. Chua, and Er. P. Li, "Numerical dispersion analysis of the unconditionally stable three-dimensional LOD-FDTD method," *IEEE Trans. Antennas Propagat.*, vol. 58, no. 12, pp. 3983-3989, Dec. 2010.

[3] Q. –F. Liu, W. –Y. Yin, Z. –Z. Chen, and P. –G. Liu, "An efficient method to reduce the numerical dispersion in the LOD-FDTD method based on the (2,4) stencil," *IEEE Trans. Antennas Propagat.*, vol. 58, no. 7, pp. 2384-2393, Jul. 2010.

[4] N. V. Kantartzis, T. Ohtani and Y. Kanai, "Accuracy-adjustable nonstandard LOD-FDTD schemes for the design of carbon nanotube interconnects and nanocomposite EMC shields," *IEEE Trans. Magn.*, vol. 49, no. 5, pp. 1821-1824, May 2013.

[5] Md. M. Rana and A. S. Mohan, "Segmented-locally-one-dimensional-FDTD method for EM propagation inside large complex tunnel environments," *IEEE Trans. Magn.*, vol. 48, no. 2, pp. 223-226, Feb. 2012.

[6] Md. M. Rana and A. S. Mohan, "Nonorthogonal LOD-FDTD method for EM scattering from two-dimensional structures," *IEEE Trans. Electromag. Compat.*, vol. 55, no. 4, pp. 764-772, Aug. 2013.